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The dependence of the emission size on the hadron mass

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Abstract

The size of the emission volume of pions and kaons, $r_{\pi\pi}$ and r_{KK} , were measured in the hadronic Z^0 decays through the two-pion and two-kaon Bose-Einstein correlations near threshold. Recently the emitter size of the two identical baryons, $\Lambda\Lambda(\bar{\Lambda}\bar{\Lambda})$, was evaluated from the dependence of the fraction of the spin $S=1$ state on the energy near threshold where it is affected by the Pauli exclusion principle. Here we show that the r dependence on the particle masses, namely the hierarchy $r_{\pi\pi} > r_{KK} > r_{\Lambda\Lambda}$ observed in the Z^0 hadronic decays, is well described in terms of the Heisenberg uncertainty principle. A good description can also be obtained via the virial theorem when applied to a general QCD potential. Other available approaches are also discussed.

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1 Introduction

It has been known for more than four decades that the correlation function of two identical particles allows us to learn about the size of the particle emission region (the Hanbury Brown and Twiss effect [1]). The interference effect originated from Bose-Einstein (Fermi-Dirac) statistics leads to an enhancement (reduction) of the two-particle correlation function which occurs when two identical particles are bosons (fermions) and have almost equal momenta. The relation to the size of the emission region is often given by the well known formula for the correlation function of two identical particles with four momenta q_i ($i = 1, 2$) and $Q = \sqrt{-(q_1 - q_2)^2}$, namely

$$\sigma_{tot} \frac{d^2\sigma_{12}}{d\sigma_1 d\sigma_2} \equiv C(Q) = 1 \pm |R(Q)|^2 , \quad (1)$$

where the sign $+$ ($-$) corresponds to bosons (fermions) and $R(Q)$ is the normalised Fourier transform of ρ , the source density [2]

$$R(Q) = \frac{\int dx \rho(x) e^{i(q_1 - q_2) \cdot x}}{\int dx \rho(x)} . \quad (2)$$

This quantity is often parametrised in terms of a “source radius” r , related to the size of the emission region, and a “chaoticity parameter” λ , which measures the strength of the effect. From Eq. (2) one derives that:

$$|R(Q)|^2 = \lambda e^{-r^2 Q^2} , \quad (3)$$

assuming a spherical emission volume with a Gaussian density distribution. The study of the Bose-Einstein Correlations (BEC) of identical boson pairs, is carried out by forming the ratio of the data correlation function to the same correlation of data or Monte Carlo (MC) sample having all the correlations, like those arising from resonances apart from those due to BEC. Since the original study of Goldhaber et al. [3] in 1959 and 1960, the BEC of pion pairs were studied in a large variety of interactions over a wide range of energies [4]. In addition, studies of BEC of kaon pairs have also been reported and values for their emitter dimension have been evaluated. The study of this dimension has recently been extended to pair of baryons, namely the $\Lambda\Lambda(\bar{\Lambda}\bar{\Lambda})$ systems using the Pauli exclusion principle. From these experimental results an interesting hierarchy of the size of the emission volumes is observed, namely $r_{\pi\pi} > r_{KK} > r_{\Lambda\Lambda}$. This volume size dependence on the hadron mass may give a new insight to the non-perturbative QCD and the hadronisation process. In this work we investigate this dependence using $\pi\pi$, KK and $\Lambda\Lambda$ pair samples present in the multi-hadron Z^0 decays. In Section 2 we summarise the current knowledge of the BEC parameters obtained from the studies of the hadronic Z^0 final states and describe our procedure to extract representative $r_{\Lambda\Lambda}$ values. In Section 3 we show that by using the Heisenberg uncertainty principle one is able to describe successfully the r dependence on the particle mass value. In Section 4 we extract the mass dependence of the size of the interaction volume which follows from the virial theorem by using a general QCD potential. Finally the information on the interaction character that can be derived from the identical particle correlations is summarised and discussed in Section 5.

2 Extraction of the two-particle emitter size

During the last decade values for $r_{\pi\pi}$ have been reported by the ALEPH [5], DELPHI [6] and OPAL [7] collaborations from measurements in the hadronic Z^0 decay events. These r values are summarised in Table 1. To notice is the fact that the r values are rather sensitive to the choice of the reference sample which was either the data $\pi^+\pi^-$ system or a sample of two-pions originating from two different events, the so called mixed event sample. The ALEPH and DELPHI collaborations have therefore opted to quote their final result as the average of the values obtained from these two methods. OPAL chose as the best r value the one obtained with a $\pi^+\pi^-$ data reference sample. The result obtained with a MC generated reference sample was used for the estimation of the systematic error. To be on equal footing with the ALEPH and DELPHI collaboration, we also present in Table 1 the average OPAL value obtained from their two methods. Finally we also list in the last row of Table 1 the over-all averaged $r_{\pi\pi}$ value of $0.74 \pm 0.01 \pm 0.14$ fm where the first error and the second one are respectively the statistical and systematic uncertainties.

Experiment	$r_{\pi\pi}$ (fm)		
	Method I	Method II	I & II Combined
ALEPH [5]	0.80 ± 0.04	0.50 ± 0.02	$0.65 \pm 0.04 \pm 0.16$
DELPHI [6]	0.82 ± 0.03	0.42 ± 0.04	$0.62 \pm 0.04 \pm 0.18$
OPAL [7]	0.96 ± 0.01	0.79 ± 0.02	$0.88 \pm 0.02 \pm 0.09$
LEP Average	0.90 ± 0.01	0.62 ± 0.01	$0.74 \pm 0.01 \pm 0.14$

Table 1: The two-pion emitter size obtained from BEC studies using the hadronic Z^0 decays carried out with two choices for the reference sample. In method I the reference sample was the data $\pi^+\pi^-$ system. In method II the reference sample was created by the event mixing technique or Monte Carlo generated events. In the last row the weighted average values are presented. The first and the second errors are respectively the statistical and systematic uncertainties. The systematic error is taken as half the difference between the values of the two methods.

Several BEC studies have been reported for the pairs of kaons. The BEC parameters obtained for the $K_S^0 K_S^0$ system do suffer from the fact that part of the observed low Q enhancement may originate from the decay of the $f_0(980) \rightarrow K^0 \bar{K}^0$. On the other hand the BEC of the $K^\pm K^\pm$ pairs are free of this problem. In the following whenever we refer to a kaon pair we mean the $K^\pm K^\pm$ systems. Using the Z^0 hadronic decays, DELPHI [8] has measured the BEC of these $K^\pm K^\pm$ pairs and obtained results for the emitter dimension r_{KK} and the strength λ_{KK} :

$$r_{KK} = 0.48 \pm 0.04(\text{stat.}) \pm 0.07(\text{syst.}) \text{ fm} \quad \text{and} \quad \lambda_{KK} = 0.82 \pm 0.11(\text{stat.}) \pm 0.25(\text{syst.})$$

A method for the measurement of the emitter dimension of two identical baryon systems in multi-hadron final state has recently been proposed by G. Alexander and H.J. Lipkin [9]. This

method, which is applicable to pairs of identical spin 1/2 baryons which decay weakly, uses the Fermi-Dirac statistics properties which suppress the over-all spin S=1 state of the identical di-baryon system near their threshold (Pauli exclusion principle).

To apply e.g. this method to $\Lambda\Lambda$ or $\Lambda\bar{\Lambda}$ pairs, the hyperon-pair is first transformed to their common centre of mass (CM) system and then each Λ decay proton is transformed to its parent hyperon CM system. The distribution of the cosine angle between these two protons, here denoted by y^* , depends on the fraction of the S=1 (or S=0) in the data. Specifically the following y^* distributions are expected for pure S=0 and S=1 states of the Λ pair

$$dN/dy^*|_{S=0} = 1 + (-1)^{B/2} \cdot \alpha_\Lambda^2 \cdot y^* \quad \text{and} \quad dN/dy^*|_{S=1} = 1 - (-1)^{B/2} \cdot \alpha_\Lambda^2 \cdot y^*/3 ,$$

where $\alpha_\Lambda=0.642\pm0.013$ [10] is the $\Lambda \rightarrow p\pi^-$ decay asymmetry parameter arising from parity violation and B is the baryon number of the di-hyperon system. These distributions are independent of the orbital angular momentum and are valid as long the Λ 's are non-relativistic in their CM system. Therefore in this method the spin composition analyses are restricted to relatively small Q values⁴. Defining ε as the fraction of the S=1 contribution to the di-hyperon pairs, it is equal to

$$\varepsilon = \frac{(S=1)}{(S=1) + (S=0)} ,$$

so that the following function:

$$dN/dy^* = f_{BG} + (1 - f_{BG}) \cdot \{(1 - \varepsilon) \cdot dN/dy^*|_{S=0} + \varepsilon \cdot dN/dy^*|_{S=1}\} ,$$

can be fitted to the data at every Q values. Here f_{BG} is the background fraction in the data. As in BEC analyses the possible, if at all, final states interactions are neglected.

The fraction of the S=1 spin content, ε , of the $\Lambda\Lambda(\bar{\Lambda}\bar{\Lambda})$ system has been measured as a function of Q in $\sim 4 \times 10^6$ hadronic Z^0 decays per experiment at LEP by the OPAL [11], ALEPH [12] and DELPHI [13] collaborations. A decrease in ε has been observed as Q approached zero in contrast to the results obtained for the $\Lambda\bar{\Lambda}$ where ε was found to be equal to 0.75 down to very low Q values. The $r_{\Lambda\Lambda}$ values deduced by the OPAL and DELPHI collaborations are :

$$\text{OPAL [11]} \quad r_{\Lambda\Lambda} = 0.19^{+0.37}_{-0.07} \pm 0.02 \text{ fm} ,$$

$$\text{DELPHI [13]} \quad r_{\Lambda\Lambda} = 0.11^{+0.05}_{-0.03} \pm 0.01 \text{ fm} .$$

The ALEPH results for the $\Lambda\Lambda$ spin composition are similar to those of OPAL and DELPHI but no attempt has been made to extract from them an $r_{\Lambda\Lambda}$ value.

The measured $\varepsilon(Q)$ values of the three experiments are compiled in Fig. 1a where the error bars are strongly dominated by the statistical uncertainties. The solid line in the same figure is the outcome of our unbinned maximum likelihood fit of the function:

$$\varepsilon(Q) = 0.75[1 - e^{-r_{\Lambda\Lambda}^2 Q^2}] , \quad (4)$$

⁴Assuming that no exotic $\Lambda\Lambda$ resonances exist in this Q region.

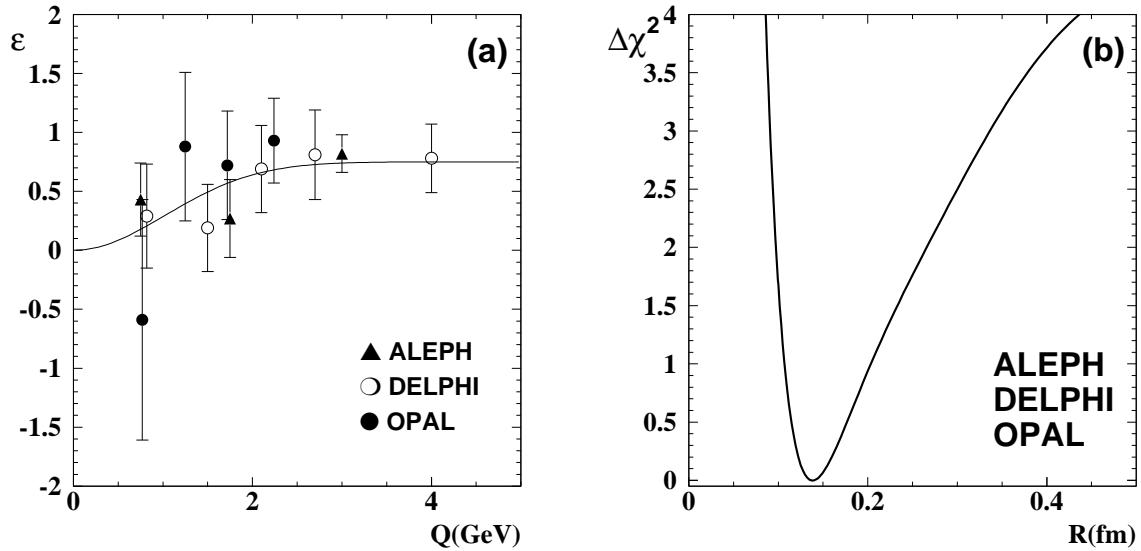


Figure 1: (a) The $S = 1$ fraction, ε , of the $\Lambda\Lambda(\bar{\Lambda}\bar{\Lambda})$ pairs measured as a function of Q by the ALEPH [12], DELPHI [13] and OPAL [11] collaborations. The solid line represents the fit results of Eq. (4) to the data points. (b) The $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ dependence on $r_{\Lambda\Lambda}$ as obtained from the fit to the combined LEP data.

to the data points plotted in the Fig 1a. The factor 0.75 appearing in Eq. (4) represents the statistical spin mixture which is proportional to $2S + 1$. From this over-all fit, with a $\chi^2/d.o.f. = 0.4$, the dimension of the $\Lambda\Lambda$ emitter and its errors are found to be

$$r_{\Lambda\Lambda} = 0.14^{+0.07}_{-0.03} \text{ fm} .$$

The $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ behaviour of this fit is shown in Fig. 1b. The ALEPH collaboration has also used an alternative method to measure $r_{\Lambda\Lambda}$. In that method one constructs, similar to the BEC studies, a correlation function of the type

$$C_2^{\Lambda\Lambda}(Q) = N_{data}^{\Lambda\Lambda}(Q)/N_{ref}^{\Lambda\Lambda}(Q) ,$$

where the numerator is the data distribution and the denominator is the distribution of a reference sample void of the Fermi-Dirac statistics. This $C_2^{\Lambda\Lambda}(Q)$ correlation is expected to decrease at low Q values due to the onset of the Pauli exclusion principle. From this analysis ALEPH evaluates $r_{\Lambda\Lambda}$ to be $(0.09 - 0.10) \pm 0.02$ fm in perfect agreement with the values obtained from the spin composition analyses.

In Fig. 2 we present the experimental results for r obtained in the analyses of the hadronic Z^0 decays. The DELPHI values, which is the only LEP experiment that measured r_{KK} , are shown by circles. The LEP averages of the measured $r_{\pi\pi}$ and $r_{\Lambda\Lambda}$ values are also shown by triangle symbols and are seen to be in good agreement with DELPHI ones. The large error associated with $r_{\pi\pi}$ reflects the systematic uncertainty due to the strong dependence of the experimental results on the choice of the reference sample. In contrast to this situation, a relative small systematic error is associated with $r_{\Lambda\Lambda}$ since in this case the spin analyses do not rely on a reference sample. In spite of the relative large errors a general non-trivial trend can be observed in the r dependence on the particle mass, namely that:

$$r_{\pi\pi} > r_{KK} > r_{\Lambda\Lambda} .$$

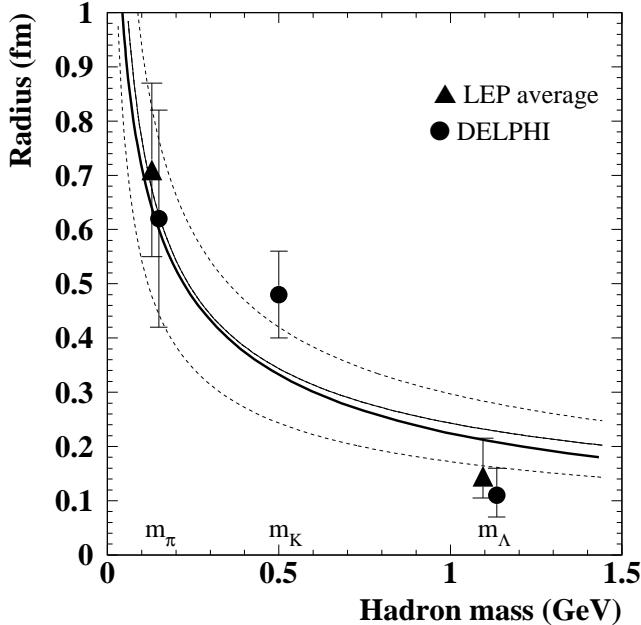


Figure 2: The emitter radius r as a function of the hadron mass m . The triangles are the DELPHI results and the circles are the averages of the measured values at LEP. For clarity the points are plotted at slightly displaced mass values. The error bars correspond to the statistical and systematic errors added in quadrature. The thin solid line represents the expectation from the Heisenberg uncertainty relations setting $\Delta t = 10^{-24}$ seconds. The upper and the lower dashed lines correspond to $\Delta t = 1.5 \times 10^{-24}$ and 0.5×10^{-24} seconds respectively. The thick solid line represents the dependence of r on m as expected from a QCD potential given by Eq. (14).

3 The emission volume size and the uncertainty relations

The maximum of the BEC enhancement of two identical bosons of mass m occurs when $Q \rightarrow 0$ which also means that the three vector momentum difference of the bosons approaches zero. This motivated the interest to link the BEC effect to the uncertainty principle [14]. From the Heisenberg uncertainty relations we have that

$$\Delta p \Delta r = 2\mu vr = m vr = \hbar c, \quad (5)$$

where m and v are the hadron mass and its velocity. Here μ is the reduced mass of the dihadron system and r is the distance between them. In Eq. (5) the momentum Δp is measured in GeV, $\Delta r \equiv r$ is given in fermi units and $\hbar c = 0.197$ GeV fm. From this, one obtains

$$r = \frac{\hbar c}{mv} = \frac{\hbar c}{p}. \quad (6)$$

Simultaneously we also use the uncertainty relation expressed in terms of time and energy

$$\Delta E \Delta t = \frac{p^2}{m} \Delta t = \hbar , \quad (7)$$

where the energy is given in GeV and Δt in seconds. Thus we have

$$p^2 = \hbar m / \Delta t \quad \text{so that} \quad p = \sqrt{\hbar m / \Delta t} . \quad (8)$$

Inserting this expression for p in Eq. (6) we finally obtain

$$r(m) = \frac{\hbar c / \sqrt{\hbar / \Delta t}}{\sqrt{m}} = \frac{c \sqrt{\hbar \Delta t}}{\sqrt{m}} . \quad (9)$$

In the following we take $\Delta t = 10^{-24}$ seconds as a representative time scale of the strong interaction which is assumed to be independent of the hadron identity and its mass. Thus $r(m) = A / \sqrt{m}$ with $A = 0.243$ fm GeV $^{1/2}$.

In Fig. 2 we show by the thin solid line the r dependence on the mass as calculated by Eq. (9) setting $\Delta t = 10^{-24}$ seconds. The sensitivity of $r(m)$ on the value of Δt is illustrated by the dashed lines in the figure. The upper and the lower dashed lines correspond respectively to the values $\Delta t = 1.5 \times 10^{-24}$ and 0.5×10^{-24} seconds. A fit of Eq. (9) to the three data points plotted in Fig. 2 yields $\Delta t = (1.2 \pm 0.3) \times 10^{-24}$ seconds with a $\chi^2/d.o.f. = 4.2$ which corresponds to a probability of $\simeq 12\%$. As seen from Fig. 2, the dependence of r on m as given in Eq. (9) describes, within errors, very well the mass dependence of the measured emitter size and offers a simple explanation for the measured hierarchy $r_{\pi\pi} > r_{KK} > r_{\Lambda\Lambda}$.

4 The emission volume size and the virial theorem

The previous section shows that the observed hierarchy $r_{\pi\pi} > r_{KK} > r_{\Lambda\Lambda}$ has a natural and a rather general explanation. Here we would like to demonstrate that such a dependence could shed light on the character of the “soft” interaction or, in other words, on the non-perturbative QCD which is responsible for the interaction at long distances. Let us continue to use the semi-classical approximation as in section 3, which means that the angular momentum

$$\ell = |\vec{p}_1 - \vec{p}_2| b_t = 2 p b_t \approx \hbar c , \quad (10)$$

where b_t is the impact parameter. To estimate the value of p we use the virial theorem [15] which yields a general connection between the average values of the kinetic and the potential energies, namely

$$2 \langle T_t \rangle = \langle \vec{b}_t \cdot \vec{\nabla}_t V(r) \rangle , \quad (11)$$

where t denotes the transverse direction. Since the motion in the transverse direction is always finite, we can safely use the virial theorem. Substituting in Eq. (11) the kinetic energy by its relation to the momentum, $T_t = p_t^2/m$, we obtain

$$\langle p_t^2 \rangle = m \langle \vec{r}_t \cdot \vec{\nabla}_t V(r) \rangle , \quad (12)$$

where we denote by r the distance between two particles with r_t being equal to $2b_t$.

From Eq. (12) one can see that the relation between $\langle p^2 \rangle$ and r depends crucially on the r dependence of the potential energy $V(r)$. Here we take $V(r)$ to be independent of the interacting particles (quarks) mass. This assumption is also in accordance with the Local Parton Hadron Duality [16] concept, which states that one can consider the production of hadron as interaction of partons, i.e. gluons and quarks, in spite of the unknown mechanism of hadronisation. This approach is supported experimentally by the single and double inclusive hadron production in $e^+ e^-$ annihilation [16]. Since the interaction between partons is independent of the produced hadron mass it follows that $\partial V(r)/\partial m = 0$. Based on this assumption and using Eq. (10) and Eq. (11), we derive an equation for the typical size of the emission volume, namely

$$r^2 \langle \vec{r} \cdot \vec{\nabla} V(r) \rangle \approx \frac{(\hbar c)^2}{m}. \quad (13)$$

To illustrate the kind of information that can be extracted from Eq. (13), we evaluate the typical size of the particle emission region from two very different potential energy $V(r)$ forms:

1. $V(r) = N_c \alpha_s \hbar c / (\pi r)$ which models the QCD induced interaction at short distances. In this case, from Eq. (13) one obtains $r \propto \hbar c / (\bar{\alpha}_S m)$.
2. $V(r) = \kappa r$ which corresponds to the confinement potential. For this potential Eq. (13) leads to $r \propto 1/m^{1/3}$.

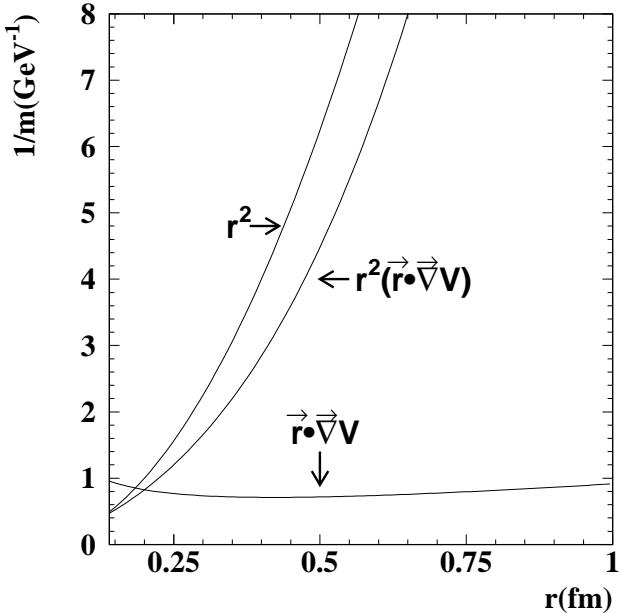


Figure 3: The behaviour of three factors, $\vec{r} \cdot \vec{\nabla} V$, $r^2(\vec{r} \cdot \vec{\nabla})$ and r^2 which contribute to the dependence of $1/m$ on the radius r when using the potential $V(r)$ defined in Eq. (14).

It is interesting to note that the experimental data plotted in Fig. 2 are well described by Eq. (13) when applied to the general potential form

$$V(r) = \kappa r - \frac{4}{3} \frac{\alpha_S \hbar c}{r}, \quad (14)$$

which is widely used to derive the wave functions and decay constants of hadrons [17]. This can be seen by comparing in Fig. 2 the thick solid line with the experimental data on the radii derived from the identical particles correlations. This line was obtained by setting Eq. (13) to an equality and evaluating Eq. (14) with the $V(r)$ parameter values of $\kappa = 0.14$ GeV $^2 = 0.70$ GeV/fm and $\alpha_s = 2\pi/9 \ln(\delta + \gamma/r)$ with $\delta = 2$ and $\gamma = 1.87$ GeV $^{-1} = 0.37$ fm obtained from the hadron wave functions and decay constants [17].

Here it should be stressed that the simple $r \propto 1/\sqrt{m}$ dependence obtained from the uncertainty principle does not contradict the virial theorem estimate. The fact that for the QCD potential defined by Eq. (14), the factor $\langle \vec{r} \cdot \vec{\nabla}_t V(r) \rangle$ is essentially constant (see Fig. 3) over the r range 0.14 to 1.0 fm, means again that $r \propto 1/\sqrt{m}$. The proportionality factor then sets the scale of distances in the non-perturbative QCD.

5 Summary and Discussion

The study reported here was motivated by the recent experimental results on the emission volume size derived from the identical particle correlations measurements. The information that this size is seen to decrease as the mass of the hadron increases, is shown here to be very valuable for the understanding of the non-perturbative QCD mechanism.

A good description of the r dependence on the hadron mass can already be obtained from the uncertainty principle. However this requires the acceptance of two, not trivial and not obvious assumptions, namely:

1. Δt does not depend on the mass value of the produced particles. As is shown below, this assumption is inconsistent with one of the models for the non-perturbative description of the confinement forces, namely, with the string approach.
2. ΔE depends only on the kinetic energy of the produced particle. It means that the interaction (potential energy) is rather small.
3. $\Delta r \sim r$ which is in the spirit with the uncertainty principle used here.

The second approach followed here is based on the virial theorem and uses the Local Parton Hadron Duality [16], which is the only argument why we can consider the potential energy to be independent of the mass of the produced particles. It looks rather impressive that the phenomenological potential, whose parameters were taken from the characteristics of the hadron wave functions and decay constants, is able to describe so successfully the experimental data. This gives us therefore the hope that the experimental data on the radius of the emission volume would be useful in any discussion of the theoretical and/or phenomenological description of the non-perturbative QCD forces responsible for confinement of quarks and gluons.

The decrease of the emitter size with the mass of the produced hadrons limits the models which can be applied to the non-perturbative QCD. For example, consider the string approach to the non-perturbative QCD, which is also the base of the Lund Model of hadronisation described in detail in [18]. In this approach the formation time which we need to create a particle with mass m , as well as the corresponding distance, are proportional to the mass of

the produced particle, namely, $t(r) \propto m/\kappa$ where κ is the string tension which appears in Eq. (14). The experimental data in Fig. 2 shows in fact the opposite trend. Therefore, the string based models may have difficulties in accommodating the data findings. On the other hand, the perturbative QCD cascade plus the Local Parton Hadron Duality [16] approach leads to $r \propto 1/m$ which reproduces qualitatively the experimental data. These two examples clearly illustrate how these measurements are able to select or reject theoretical proposals. For this reason more accurate measurements of $r_{\pi\pi}$, r_{KK} and $r_{\Lambda\Lambda}$ should be very valuable.

We would like to stress that the observed hierarchy of the sizes of the emission volume is providing us an additional information on the dynamics of interaction to that given by the well established experimentally dependence of the effective source sizes on the produced particles transverse mass, m_t . Experiments with heavy-ions and hadron-hadron reactions as well as LEP studies [19] show that $r \sim 1/\sqrt{m_t}$. In these experiments only Bose-Einstein correlations between identical pions have been measured, where r is considered to be proportional to the production time [20]. It should be stressed that this production time is defined as the invariant time elapsed between the creation of a $q\bar{q}$ pair and the formation of a pion. For large p_t an hypothesis has been suggested that the production time is larger than the formation time. This hypothesis is able to reproduce the experimental dependence of r on m_t . It is reasonable to assume that the dependence of the size of the source on the mass of the produced particle is related to the formation time rather than to the production one. This is a reason why such experiments as discussed here can give a new insight to the nonperturbative dynamics of hadron production.

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